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$\frac{88}{89}$

Final Exam: MTH 221, Fall 2016

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QUESTION 1. (9 pts) Let $D = span\{\underbrace{(1,0,0,-1)}_{Q_1}, \underbrace{(1,2,0,1)}_{Q_2}, \underbrace{(-3,1,-1,2)}_{Q_3}\}$. Use Gram-Schmidt algorithm to construct an **orthogonal** basis for D. (Brief lecture: Assume that $\{w_1, w_2, w_3\}$ is an orthogonal basis for D, then $\{w_1/||w_1||, w_2/||w_2||, w_3/||w_3||\}$ is called an **orthonormal** basis for D, but here just find an orthogonal basis for D).

$$w_1 = Q_1 = (1,0,0,-1)$$

$$w_2 = Q_2 - \frac{Q_2 \cdot w_1}{||w_1||^2} w_1$$

$$w_3 = Q_3 - \frac{Q_3 \cdot w_2}{||w_2||^2} w_2 - \frac{Q_3\, w_1}{||w_1||^2}\, w_1$$

$\frac{9}{9}$

$$Q_2 \cdot w_1 = (1,2,0,1)\cdot(1,0,0,-1) = 1+0+0-1 = 0$$

$$||w_1||^2 = w_1 \cdot w_1 = (1,0,0,-1)\cdot(1,0,0,-1) = 1+1 = 2$$

$$w_2 = Q_2 - \frac{0}{2} w_1 = Q_2 = (1,2,0,1)$$

$$Q_3 \cdot w_2 = (-3,1,-1,2)\cdot(1,2,0,1) = -3+2+0+2 = 1$$

$$||w_2||^2 = w_2 \cdot w_2 = (1,2,0,1)\cdot(1,2,0,1) = 1+4+0+1 = 6$$

$$Q_3 \cdot w_1 = (-3,1,-1,2)(1,0,0,-1) = -3+0+0-2 = -5$$

$$w_3 = Q_3 - \frac{1}{6} w_2 - \left(\frac{-5}{2}\right) w_1$$

$$w_3 \overset{\times 6}{=} \left[(-3,1,-1,2) - \frac{1}{6}(1,2,0,1) + \frac{5}{2}(1,0,0,-1)\right]$$

$$w_3 = (-18,6,-6,12) - (1,2,0,1) + (15,0,0,-15)$$

$$w_3 = (-4,4,-6,-4)$$

orthogonal basis: $\{(1,0,0,-1), (1,2,0,1), (-4,4,-6,-4)\}$ ✓ ✓ ✓

checking: $w_1 \cdot w_2 = 1-1 = 0$ ✓

$w_2 \cdot w_3 = -4+8-4 = 0$ ✓

$w_1 \cdot w_3 = -4+4 = 0$ ✓

QUESTION 2. (9 pts) Find the solution set to the following system (Show the work).

$$\begin{aligned} x_1 + 2x_2 + 4x_3 + 4x_4 &= 1, \\ 2x_1 + 5x_2 + 9x_3 + 4x_4 &= 0, \\ x_2 + x_3 &= 1. \end{aligned}$$

$$\begin{array}{c} \quad x_1 \quad x_2 \quad x_3 \quad x_4 \\ \left[\begin{array}{cccc|c} 1 & 2 & 4 & 4 & 1 \\ 2 & 5 & 9 & 4 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{array}\right] \end{array} \xrightarrow{-2R_1+R_2 \to R_2} \left[\begin{array}{cccc|c} 1 & 2 & 4 & 4 & 1 \\ 0 & 1 & 1 & -4 & -2 \\ 0 & 1 & 1 & 0 & 1 \end{array}\right] \xrightarrow[-R_2+R_3 \to R_3]{-2R_2+R_1 \to R_1}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 12 & 5 \\ 0 & 1 & 1 & -4 & -2 \\ 0 & 0 & 0 & 4 & 3 \end{array}\right] \xrightarrow{\frac{1}{4}R_3} \left[\begin{array}{cccc|c} 1 & 0 & 2 & 12 & 5 \\ 0 & 1 & 1 & -4 & -2 \\ 0 & 0 & 0 & 1 & \frac{3}{4} \end{array}\right] \xrightarrow[-12R_3+R_1 \to R_1]{4R_3+R_2 \to R_2}$$

$$\begin{array}{c} x_1 \; x_2 \; x_3 \; x_4 \\ \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & -4 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & \frac{3}{4} \end{array}\right] \end{array}$$

$x_1 + 2x_3 = -4 \longrightarrow x_1 = -2x_3 - 4$

$x_2 + x_3 = 1 \longrightarrow x_2 = 1 - x_3$

$x_4 = \frac{3}{4}$

solution set: $\left\{ \left(-2x_3 - 4,\ 1 - x_3,\ x_3,\ \frac{3}{4}\right) \mid x_3 \in \mathbb{R} \right\}$

checking: $x_3 = 0$ $\left(-4, 1, 0, \frac{3}{4}\right)$

$-4 + 2 + 3 = 1$ ✓

$-8 + 5 + 3 = 0$ ✓

$1 + 0 = 1$ ✓

$x_3 = 1$ $\left(-6, 0, 1, \frac{3}{4}\right)$

$-6 + 4 + 3 = 1$ ✓

$-12 + 9 + 3 = 0$ ✓

$0 + 1 = 1$ ✓

9/9

QUESTION 3. (4 pts) You are given the below system

$$\begin{aligned} x + y &= 1 \\ y + z &= 2 \\ 2x + bz &= a. \end{aligned}$$

(i) For what values of b, a will the system have unique solution? show your solution

$$\begin{array}{c} x\ \ y\ \ z \\ \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 0 & b & a \end{array}\right] \end{array} \xrightarrow{-2R_1+R_3\to R_3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & -2 & b & a-2 \end{array}\right] \xrightarrow[2R_2+R_3\to R_3]{-R_2+R_1\to R_1} \left[\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & b+2 & a+2 \end{array}\right]$$

$\boxed{\text{unique solution: if } b \neq -2 \text{ and } a \in \mathbb{R}}$

2/2

(ii) For what values of b, a will the system be inconsistent? (i.e., the system has no solution) show your solution

if $b = -2$ and $a \neq 2$

2/2

because in that case we will have

$0 = a$ number

which is impossible which would make the system inconsistent.

ex $a = 2$ $\left[\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 2 \end{array}\right] \to 0 = 2$

QUESTION 4. (3 pts) Let $Q_1 = (a, 1, 1)$, $Q_2 = (0, a, 1)$, and $Q_3 = (a-1, 1, 1)$. For what values of a will Q_1, Q_2, and Q_3 be independent? show your solution

$$A = \begin{bmatrix} a & 1 & 1 \\ 0 & a & 1 \\ a-1 & 1 & 1 \end{bmatrix} \xrightarrow{-R_1+R_3\to R_3} \begin{bmatrix} a & 1 & 1 \\ 0 & a & 1 \\ -1 & 0 & 0 \end{bmatrix}$$

$$|A| = (-1)^{3+1}(-1)\begin{vmatrix} 1 & 1 \\ a & 1 \end{vmatrix} = -1(1-a) = -1+a = a-1$$

if $|A| \neq 0 \to Q_1, Q_2, Q_3$ independent

therefore if $a \neq 1$, the points are independent

3/3

checking if |A| was right:

$$|A| = (-1)^{2+2} a \begin{vmatrix} a & 1 \\ a-1 & 1 \end{vmatrix} + (-1)^{3+2} \begin{vmatrix} a & 1 \\ a-1 & 1 \end{vmatrix} = a(a-(a-1)) - [a-(a-1)] = a(\cancel{a}-\cancel{a}+1) - (a-a+1) = a-1$$

QUESTION 5. (3 pts) Let $T : R^2 \to R$ be a linear transformation such that $T(2,1) = 5$ and $T(-1,2) = 3$. Find $T(3,-1)$.

$\alpha_1(2,1) + \alpha_2(-1,2) = (3,-1)$

$\times 2 \begin{cases} 2\alpha_1 - \alpha_2 = 3 \\ \alpha_1 + 2\alpha_2 = -1 \end{cases}$ $\begin{cases} 4\alpha_1 - \cancel{2\alpha_2} = 6 \\ \alpha_1 + \cancel{2\alpha_2} = -1 \end{cases}$

$5\alpha_1 = 5$

$\boxed{\alpha_1 = 1}$ $\boxed{\alpha_2 = -1}$

$\Longrightarrow 1(2,1) - 1(-1,2) = (3,-1)$

$T(2,1) - T(-1,2) = T(3,-1)$

$5 - 3 = 2$

$\boxed{T(3,-1) = 2}$

3/3

QUESTION 6. (3 pts) Give me two meanings to the statement $Q_1, Q_2, \ldots, Q_k$ are dependent points in R^n.

One meaning is : dimension of span $\{Q_1, Q_2, \ldots, Q_k\}$ is less than k.

Second meaning is : determinant of $\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_k \end{bmatrix}$ is zero. and $\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_k \end{bmatrix}$ is not invertible.

3/3

So: At least one of the points can be written as a linear combination of the others.

QUESTION 7. **(10 pts)** Consider the matrix $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

i) **(5 pts)** Find A^{-1}

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ -1 & 2 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \overset{R_1+R_2 \to R_2}{\sim} \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ 0 & 3 & -2 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \overset{\frac{1}{3}R_2}{\sim}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \underset{R_2+R_3 \to R_3}{\overset{-R_2+R_1 \to R_1}{\sim}} \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{-4}{3} & \frac{2}{3} & \frac{-1}{3} & 0 \\ 0 & 1 & \frac{-2}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 1 \end{array}\right] \underset{2R_3+R_2 \to R_2}{\overset{4R_3+R_1 \to R_1}{\sim}}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 4 \\ 0 & 1 & 0 & 1 & 1 & 2 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 1 \end{array}\right] \overset{3R_3}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 4 \\ 0 & 1 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 & 1 & 3 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 1 & 2 \\ 1 & 1 & 3 \end{bmatrix}$$

✓ 5/5

ii) **(5 pts)** Find the matrix B such that $A + AB = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ -1 & 1 & 0 \end{bmatrix}$

$$A\left[I_3 + B\right] = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ -1 & 1 & 0 \end{bmatrix} \to A^{-1}A\left[I_3 + B\right] = A^{-1}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ -1 & 1 & 0 \end{bmatrix}$$

$$\left[I_3 + B\right] = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 1 & 2 \\ 1 & 1 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -3 & 6 & 4 \\ -1 & 3 & 3 \\ -2 & 4 & 3 \end{bmatrix}$$

$$I_3 + B = \begin{bmatrix} -3 & 6 & 4 \\ -1 & 3 & 3 \\ -2 & 4 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} -3 & 6 & 4 \\ -1 & 3 & 3 \\ -2 & 4 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 6 & 4 \\ -1 & 2 & 3 \\ -2 & 4 & 2 \end{bmatrix}$$

5/5

QUESTION 8. **(10 pts)** Let $A = \begin{bmatrix} 1 & 3 & 1 & 2 \\ 0 & -1 & 5 & -1 \\ 2 & 6 & 2 & 4 \\ 0 & 1 & -5 & 1 \end{bmatrix} \overset{-2R_1+R_3 \to R_3}{\sim} \begin{bmatrix} 1 & 3 & 1 & 2 \\ 0 & -1 & 5 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -5 & 1 \end{bmatrix} \overset{R_2+R_4 \to R_4}{\sim} \begin{bmatrix} 1 & 3 & 1 & 2 \\ 0 & -1 & 5 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

i) **(4 pts)** Find a basis and the dimension for the $Row(A)$ and for the $Col(A)$.

Basis for Row (A) : $\{(1,3,1,2), (0,-1,5,-1)\}$ ✓

Basis for Col (A) : $\{(1,0,2,0), (3,-1,6,1)\}$ ✓

Dimension of Row (A) = Dimension of Col (A) = 2 ✓ 4/4

ii) **(2 pts)** What is the dimension of the null-space of A ? (i.e., find $dim(N(A))$). (You do not have to calculate the null-space to answer this question).

dim (N (A)) = number of free variables = 2

iii) **(4 pts)** does $(8, -2, 14, 2)$ belong to $Col(A)$? Show the work

$\alpha_1 (1,0,2,0) + \alpha_2 (3,-1,6,1) = (8,-2,14,2)$

$\alpha_1 + 3\alpha_2 = 8 \longrightarrow \alpha_1 + 6 = 8 \longrightarrow \boxed{\alpha_1 = 2}$

$-\alpha_2 = -2 \longrightarrow \alpha_2 = 2$

$2\alpha_1 + 6\alpha_2 = 14 \qquad 2(2) + 6(2) = 4 + 12 = 16$

$2\alpha_1 + 12 = 14 \longrightarrow 2\alpha_1 = 14 - 12 = 2 \quad \boxed{\alpha_1 = 1}$

$\alpha_2 = 2$

we do not get a unique value for α_1

$\Longrightarrow (8,-2,14,2)$ cannot be written as a linear combination of the column space basis

$\Longrightarrow (8,-2,14,2)$ does not belong to Col (A).

4/4

QUESTION 9. **(3 points)** Assume that $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ such that $|A| = 4$. Let $B = \begin{bmatrix} a+d & b+e & c+f \\ 2g & 2h & 2i \\ -d & -e & -f \end{bmatrix}$ Find $|2A^{-1}B|$.

$|A^{-1}| = \frac{1}{4}$

$|2A^{-1}||B| =$

$2^3 \cdot \frac{1}{4} \cdot 8 = 2^3 \cdot 2 = 2^4$

$= 16$ ✓

3/3

$\overset{A}{\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}} \overset{R_2 \leftrightarrow R_3}{\sim} \overset{C}{\begin{bmatrix} a & b & c \\ g & h & i \\ d & e & f \end{bmatrix}} \overset{R_3+R_1 \to R_1}{\sim} \overset{D}{\begin{bmatrix} a+d & b+e & c+f \\ g & h & i \\ d & e & f \end{bmatrix}}$

$\overset{-R_3}{\sim} \underset{E}{\begin{bmatrix} a+d & b+e & c+f \\ g & h & i \\ -d & -e & -f \end{bmatrix}} \overset{2R_2}{\sim} \underset{B}{\begin{bmatrix} a+d & b+e & c+f \\ 2g & 2h & 2i \\ -d & -e & -f \end{bmatrix}}$

$|C| = -|A| = -4$

$|D| = |C| = -4$

$|E| = -|D| = 4$

$|B| = 2|E| = 8$

QUESTION 10. (8 pts) Find a basis for the kernel of T (Ker(T)), where T is the linear transformation $T : P_5 \to R^{2\times 2}$ defined by

$$T(ax^4 + bx^3 + cx^2 + dx + e) = \begin{bmatrix} a+b & b-c \\ c-e & e-d \end{bmatrix}$$

$a+b=0 \quad a=-b$

$b-c=0 \quad b=c$

$c-e=0 \quad c=e$

$e-d=0 \quad e=d$

Kernel : $\{-bx^4+bx^3+bx^2+bx+b \mid b\in\mathbb{R}\}$

$= \{b(-x^4+x^3+x^2+x+1) \mid b\in\mathbb{R}\}$

$= \text{span}\{-x^4+x^3+x^2+x+1\}$

basis : $\{-x^4+x^3+x^2+x+1\}$

8/8

i) (1 point) Is T as above ONTO? Explain Yes. $\dim(\ker(T)) = 1$

$\boxed{\dim(\ker(T)) + \dim(\text{Image}(T)) = \dim(\text{domain})}$

therefore : $1 + \dim(\text{Image}) = 5 \to \dim(\text{Image}) = 4$

$\dim(\text{co-domain}) = 4 = \dim(\text{Image}) \Longrightarrow T$ is ONTO

ii) (1 point) Is T as above one-to-one? Explain.

No. $\dim(\ker(T)) = 1$

T is only 1-1 if $\dim(\ker(T)) = 0$

QUESTION 11. (12 pts) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$AQ^T = \alpha Q^T \qquad AQ^T - \alpha Q^T = 0 \qquad (A - \alpha I_3) Q^T = 0$

$\alpha I_3 - A$

i) **(4 pts)** Find all eigenvalues of A.

$$\alpha I_3 - A = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} \alpha - 1 & -1 & -1 \\ 0 & \alpha - 2 & 0 \\ 0 & 0 & \alpha - 2 \end{bmatrix}$$

$$C_A(\alpha) = (\alpha - 1)(\alpha - 2)(\alpha - 2) = (\alpha - 2)^2 (\alpha - 1)$$

eigen values : 2, 2, 1

ii) **(5 pts)** For each eigenvalue b of A, find the corresponding eigenspace E_b.

$$E_1 = N \begin{bmatrix} 0 & -1 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & -1 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{-R_1 + R_2 \to R_2} \begin{bmatrix} 0 & -1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow[R_2 + R_1 \to R_1]{R_2 + R_3 \to R_3} \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_2 = 0$

$x_3 = 0$

$x_1 \in \mathbb{R}$

$E_1 = \{(x_1, 0, 0) \mid x_1 \in \mathbb{R}\}$

$= \text{span}\{(1, 0, 0)\}$

dim = 1

$$E_2 = N \begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 - x_2 - x_3 = 0$

$x_1 = x_2 + x_3$

$E_2 = \{(x_2 + x_3, x_2, x_3) \mid x_2, x_3 \in \mathbb{R}\}$

$E_2 = \{x_2(1, 1, 0) + x_3(1, 0, 1) \mid x_2, x_3 \in \mathbb{R}\}$

$E_2 = \text{span}\{(1, 1, 0), (1, 0, 1)\}$

dim = 2

12/12

iii) **(3 pts)** Is A diagonalizable? If so, find an invertible matrix Q and a diagonal matrix D such that $A = QDQ^{-1}$.

Yes.

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad Q = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

QUESTION 12. (10 pts) Determine whether W is a subspace of V, if yes give a mathematical argument to prove it and if no then give me an example that illustrates that one of the three axioms fail

i) **(3 pts)** $W = \{(a, b, c, d) \in R^4 \mid ac = bd\}$

Axiom 2: addition

take $(1, 2, 8, 4) \in W$ (a, b, c, d)

$ac = 8 \quad bd = 8$

take $(3, 2, 4, 6) \in W$ (a, b, c, d)

$ac = 12 \quad bd = 12$

$(1,2,8,4) + (3,2,4,6) = (4, 4, 12, 10) \notin W$

$ac = 48$
$bd = 40$ X

axiom fails $\Longrightarrow$ not a subspace

3/3

ii) **(3 pts)** $W = \{f(x) \in P_3 \mid \text{Degree}(f(x)) = 2\}$

$\{a_2x^2 + a_1x + a_0 \mid \text{Degree}(f(x)) = 2\} \to a_2 \neq 0 \to$ First axiom fails

Second axiom:

$f_1 = -2x^2 + x + 1 \in W$
$f_2 = 2x^2 + x + 1 \in W$

$f_1 + f_2 = 2x + 2 \notin W$

2nd axiom fails

$\Longrightarrow$ not a subspace

3/3

iii) **(4 pts)** Let A be a fixed 3×3 matrix and $W = \{Q = (a, b, c) \in R^3 \mid AQ^T = 3Q^T\}$. We are assuming that α is an eigen value of A

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

W is the Eigen Space of the Eigen value $\underline{3}$ of A. Eigen spaces can be written as span therefore they are subspaces.

~~X~~

$\alpha I_3 - A = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} \alpha-1 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha-3 \end{bmatrix}$

$E_3 = N\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $x_1 = 0$, $x_2 = 0$, $x_3 \in \mathbb{R}$ $E_3 = \text{Span}\{(0,0,1)\}$

QUESTION 13. (3 points) Imagine that I just told you that by using the least square method, the best "fit" plane of the form $z = ax + by$ to the points $(1, 1, 1)$, $(-1, 1, -1)$, and $(0, 2, 6)$ is $z = x + 2y$. What does that mean? explain the meaning of the answer by doing the actual calculation.

(Q_1, Q_2, Q_3)

It means that $|PQ_1|^2 + |PQ_2|^2 + |PQ_3|^2$ is minimum

meaning: We found a plane of the form $z = ax + by$ (P) which has the least distance possible from the three points Q_1, Q_2, Q_3
(squared)

$z = x + 2y$
$\begin{cases} x = 1 \quad y = 1 \\ x = -1, y = 1 \\ x = 0, y = 2 \end{cases}$ $\begin{cases} z = 1 + 2 = 3 \\ z = -1 + 2 = 1 \\ z = 2 \times 2 = 4 \end{cases}$

$|PQ_1|^2 + |PQ_2|^2 + |PQ_3|^2 =$

$(1-3)^2 + (-1-1)^2 + (6-4)^2 =$

$(-2)^2 + (-2)^2 + (2)^2 = 4 + 4 + 4 = 12$

This distance (12) is minimum. ie, we cannot find a plane of the form $z = ax + by$ where $|PQ_1|^2 + |PQ_2|^2 + |PQ_3|^2$ is less than 12.

Faculty information

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